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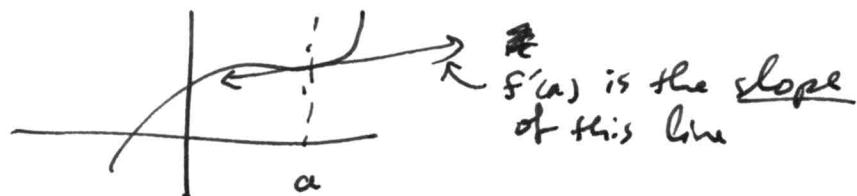
Derivatives

1. Write down the limit definition of the derivative $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

2. Explain the graphical meaning of the derivative $f'(x)$.

$f'(a)$ ~~is the slope of $f(x)$ at a~~ = the instantaneous rate of change of $f(x)$ at a



3. Write down an equation for the tangent line to $f(x)$ at the point $(a, f(a))$.

$$y = f'(a)(x-a) + f(a)$$

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4. Compute the derivative of the following function:

$$f(x) = 25x^5 + 32x^4 + 9x^3 + 144$$

$$f'(x) = \frac{d}{dx} (25x^5 + 32x^4 + 9x^3 + 144)$$

$$= 125x^4 + 128x^3 + 27x^2 + 0$$

$$\begin{array}{r} 25 \\ \times 5 \\ \hline 125 \end{array}$$

$$\begin{array}{r} 32 \\ \times 4 \\ \hline 128 \end{array}$$

5. Compute the derivative of the following function:

$$f(x) = \sqrt{x} - 7e^x + (-1) \cdot x^9$$

$$f'(x) = \frac{d}{dx} [x^{\frac{1}{2}}] - 7 \frac{d}{dx} [e^x] - \frac{d}{dx} [x^9]$$

$$= \frac{1}{2}x^{-\frac{1}{2}} - 7e^x - 9x^8 = \boxed{\frac{1}{2\sqrt{x}} - 7e^x - 9x^8}$$

6. Compute the derivative of the following function:

$$(fg)' = fg' + gf'$$

$$f(x) = (x^2 + x + 1) \cdot e^x$$

$$f'(x) = \frac{d}{dx} [(x^2 + x + 1) \cdot e^x] = (x^2 + x + 1) \cdot \frac{d}{dx} [e^x] + e^x \cdot \frac{d}{dx} [x^2 + x + 1]$$

$$= (x^2 + x + 1) \cdot e^x + e^x \cdot (2x + 1)$$

$$= \boxed{e^x (x^2 + 3x + 2)}$$

7. Compute the derivative of the following function:

$$f(x) = e^x \cdot (\sqrt{x} + 1)$$

$$f'(x) = \frac{d}{dx} [e^x \cdot (x^{\frac{1}{2}} + 1)] = e^x \cdot \frac{d}{dx} (x^{\frac{1}{2}} + 1) + (x^{\frac{1}{2}} + 1) \cdot \frac{d}{dx} [e^x]$$

$$= e^x \cdot \left(\frac{1}{2}x^{-\frac{1}{2}} \right) + (x^{\frac{1}{2}} + 1) \cdot e^x$$

$$= \boxed{e^x \left(\frac{1}{2\sqrt{x}} + \sqrt{x} + 1 \right)}$$

8. Compute the derivative of the following function:

$$f(x) = 2e^x \cdot \ln(x)$$

$$f'(x) = \frac{d}{dx} [2e^x \cdot \ln(x)] = 2e^x \cdot \frac{d}{dx} [\ln(x)] + \ln(x) \cdot \frac{d}{dx} [2e^x]$$

$$= 2e^x \cdot \frac{1}{x} + \ln(x) \cdot 2e^x$$

$$= \boxed{2e^x \left(\frac{1}{x} + \ln(x) \right)}$$

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9. Compute the derivative of the following function:

$$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$$

$$f(x) = \frac{e^x}{x}$$

$$f'(x) = \frac{x \cdot \frac{d}{dx}[e^x] - e^x \cdot \frac{d}{dx}[x]}{x^2} = \frac{x \cdot e^x - e^x}{x^2}$$

10. Compute the derivative of the following function:

$$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$$

$$f(x) = \frac{x^2 + 1}{x^2 + x + 1}$$

$$f'(x) = \frac{(x^2 + x + 1) \cdot \frac{d}{dx}[x^2 + 1] - (x^2 + 1) \cdot \frac{d}{dx}[x^2 + x + 1]}{(x^2 + x + 1)^2}$$

$$= \frac{(x^2 + x + 1)(2x) - (x^2 + 1) \cdot (2x + 1)}{(x^2 + x + 1)^2}$$

11. Compute the derivative of the following function:

$$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$$

$$f(x) = \frac{e^x + 1}{e^x - 1}$$

$$f'(x) = \frac{(e^x - 1) \cdot \frac{d}{dx}(e^x + 1) - (e^x + 1) \cdot \frac{d}{dx}(e^x - 1)}{(e^x - 1)^2}$$

$$= \frac{(e^x - 1) \cdot e^x - (e^x + 1) \cdot e^x}{(e^x - 1)^2} = \frac{e^x(e^x - 1 - e^x - 1)}{(e^x - 1)^2} = \boxed{\frac{-2e^x}{(e^x - 1)^2}}$$

12. Compute the derivative of the following function:

$$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$$

$$f(x) = \frac{x}{\ln(x)}$$

$$f'(x) = \frac{\ln(x) \frac{d}{dx}[x] - x \cdot \frac{d}{dx}[\ln(x)]}{(\ln(x))^2}$$

$$= \frac{\ln(x) - x \cdot \frac{1}{x}}{(\ln(x))^2} = \boxed{\frac{\ln(x) - 1}{(\ln(x))^2}}$$

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The Chain Rule

$$\frac{d}{dx} [g(f(x))] = g'(f(x)) \cdot f'(x)$$

1. Let
- $f(x) = (1 - 2x)^4$
- . Find
- $f'(x)$
- .

$$f'(x) = \frac{d}{dx} \left[(1-2x)^4 \right]$$

$g(u) = u^4 \Rightarrow g'(u) = 4u^3$
 $f(x) = 1-2x \Rightarrow f'(x) = -2$

$$= 4(1-2x)^3(-2x)$$

2. Let
- $f(x) = \frac{1}{x-x^2}$
- . Find
- $f'(x)$
- .

$$f'(x) = \frac{d}{dx} \left[(x-x^2)^{-1} \right]$$

$g(u) = u^{-1} \Rightarrow g'(u) = (-1)u^{-2}$
 $f(x) = x-x^2 \Rightarrow f'(x) = 1-2x$

$$= (-1)(x-x^2)^{-2} \cdot (1-2x) = \frac{2x-1}{(x-x^2)^2}$$

3. Let
- $C(x) = \sqrt{0.1x^2 - 2x + 10}$
- . Find
- $\frac{dC}{dx}$
- .

$$\frac{dC}{dx} = \frac{d}{dx} \left[(0.1x^2 - 2x + 10)^{\frac{1}{2}} \right]$$

$g(u) = u^{\frac{1}{2}} \Rightarrow g'(u) = \frac{1}{2}u^{-\frac{1}{2}}$
 $f(x) = 0.1x^2 - 2x + 10 \Rightarrow f'(x) = 0.2x - 2$

$$= \frac{1}{2}(0.1x^2 - 2x + 10)^{-\frac{1}{2}} \cdot (0.2x - 2)$$

4. Let
- $y(x) = \frac{(2x-1)^2}{(1-x)^3}$
- . Find
- $\frac{dy}{dx}$
- .

Outermost thing is quotient
⇒ use quotient rule

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left[\frac{(2x-1)^2}{(1-x)^3} \right] \\ &= \frac{(1-x)^3 \cdot \frac{d}{dx}[(2x-1)^2] - (2x-1)^2 \cdot \frac{d}{dx}[(1-x)^3]}{(1-x)^6} \\ &\quad \rightarrow \frac{d}{dx}[(1-x)^3] \\ &\quad \begin{cases} g(u) = u^3 \\ f(u) = 1-x \end{cases} \\ &= \frac{(1-x)^3 \cdot 2(2x-1) \cdot 2 - (2x-1)^2 \cdot 3(1-x)^2(-1)}{(1-x)^6} \\ &= \frac{2(2x-1) \cdot 2 - (2x-1)^2 \cdot 3(1-x)^2(-1)}{(1-x)^4} \\ &= \dots \end{aligned}$$

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5. Let $f(x) = \ln(1 - 2x)$. Find $f'(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx} [\ln(1-2x)] \\ g(u) &= \ln(u) \Rightarrow g'(u) = \frac{1}{u} \\ f(x) &= 1-2x \Rightarrow f'(x) = -2 \\ &= \frac{1}{1-2x} \cdot (-2) = \frac{-2}{1-2x} = \frac{2}{2x-1} \end{aligned}$$

6. Let $y = e^{2x+1}$. Find $\frac{dy}{dx}$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [e^{2x+1}] \\ g(u) &= e^u \Rightarrow g'(u) = e^u \\ f(x) &= 2x+1 \Rightarrow f'(x) = 2 \\ &= e^{2x+1} \cdot 2 \end{aligned}$$

7. Let $y(x) = e^{x \cdot \ln(x)}$. Find $\frac{dy}{dx}$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [e^{x \cdot \ln(x)}] \\ g(u) &= e^u \Rightarrow g'(u) = e^u \\ f(x) &= x \cdot \ln(x) \Rightarrow f'(x) = \frac{d}{dx} [x \cdot \ln(x)] = x \cdot \frac{1}{x} + \ln(x) \cdot 1 = 1 + \ln(x) \\ &= e^{x \cdot \ln(x)} (1 + \ln(x)) \end{aligned}$$

use product rule.

8. Let $y = 2^x$. Find $y'(x)$.

$$y' = \frac{d}{dx} [2^x] = 2^x \cdot \ln(2)$$

9. Let $F(t) = 1000 \cdot (1.07)^{12t}$. Find $\frac{dF}{dt}$.

$$\begin{aligned} \frac{dF}{dt} &= \frac{d}{dt} [1000 \cdot (1.07)^{12t}] = 1000 \cdot \frac{d}{dt} [(1.07)^{12t}] \\ g(u) &= (1.07)^u \Rightarrow g'(u) = (1.07)^u \cdot \ln(1.07) \\ f(t) &= 12t \Rightarrow f'(t) = 12 \end{aligned}$$

$$= 1000 \cdot \underbrace{(1.07)^{12t}}_{g'(f(t))} \cdot \underbrace{\ln(1.07)}_5 \cdot \underbrace{12}_{f'(t)} = \dots$$

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Tangents

$$\text{Formula for a line: } y = m(x - x_1) + y_1$$

1. Find the equation of the line tangent to $f(x) = 6x \cdot e^x$ at the point with $x = 0$.

need $m = \text{slope to } f \text{ at } 0 = f'(0)$.

$$\boxed{x_1 = 0 \\ \Rightarrow y_1 = f(0) = 0}$$

first compute $f'(x) = \frac{d}{dx}[6x \cdot e^x]$

$$= 6x \cdot \frac{d}{dx}[e^x] + e^x \cdot \frac{d}{dx}[6x]$$

$$f'(x) = 6x e^x + 6 e^x$$

$$m = f'(0) = 6 \cdot 0 \cdot e^0 + 6 \cdot e^0 = 6$$

$\Rightarrow y = 6(x - 0) + 0 = 6x$ is the tangent line

2. Find the equation of the line tangent to $f(x) = \frac{\ln(x)}{x}$ at the point $(1, 0)$

$$f'(x) = \frac{d}{dx}\left[\frac{\ln(x)}{x}\right] = \frac{x \cdot \frac{d}{dx}[\ln(x)] - \ln(x) \cdot \frac{d}{dx}[x]}{x^2}$$

$$= \frac{x \cdot \frac{1}{x} - \ln(x)}{x^2} = \frac{1 - \ln(x)}{x^2} = f'(x)$$

slope of tangent at 1 = $f'(1) = \frac{1 - \ln(1)}{1^2} = \frac{1 - 0}{1} = 1$

tangent = $y = 1(x - 1) + 0 = x - 1$

3. Find the equation of the line tangent to $f(x) = (2x + 1)(5 - x)$ at 2.

$$f(x) = 10x - 2x^2 + 5 - x = -2x^2 + 9x + 5$$

$$f'(x) = -4x + 9$$

$$x_1 = 2$$

$$y_1 = f(2) = (2 \cdot 2 + 1)(5 - 2) = 5 \cdot 3 = 15$$

$$m = f'(2) = -4 \cdot 2 + 9 = -8 + 9 = 1$$

tangent line = $y = 1(x - 2) + 15$